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# NON-LINEAR PERFECT RECONSTRUCTION FILTER BANKS FOR IMAGE CODING

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## ABSTRACT

This paper presents a new architecture for designing non-linear, critically decimated, perfect reconstruction filter banks. It is believed that such filters have a wide range of applications including that of still image and image sequence compression.

Example filters are presented, for still image compression using a quincunx system. These filters are shown to out-perform other linear and non-linear methods, both in terms of subjective and objective image quality.

## 1. INTRODUCTION

Figure 1 shows the usual structure for a 2-band, critically decimated filter bank. Perfect reconstruction is achieved if (in the absence of quantization), the output of the system is identical to the input ( $\hat{X} = X$ ). The 2-band system is often applied recursively to the low band, in order to provide a 'wavelet' decomposition. In recent years, many methods have been proposed for the design of linear, perfect reconstruction filter banks (e.g. [1, 2, 3]).

For image and video coding [4], quantization is usually applied to the sub-band signals, which causes quantization errors to occur in the decoded image. With linear filters, these errors are most visible around sharp edges, where they appear as ringing artifacts, (Gibbs phenomenon). Even with linear filters designed to reduce ringing [2], there is still significant noise around sharp edges.

Egger et al. [5] showed that the 2-band, critically decimated, perfect reconstruction conditions can be satisfied using non-linear half-band filters, shown in figure 2. This system is actually equivalent to interpolative coding schemes, commonly used for loss-less im-

age compression [6]. This method produces a 'low-pass' sub-band by sub-sampling the image (without filtering). The 'high pass' sub-band is then formed by subtracting a prediction found by interpolating from the 'low-pass' sub-band. Although these non-linear systems can give better subjective results, the lack of any low-pass analysis filter is overly restrictive and can limit compression (by introducing aliasing). In order to combat this, Egger et al. [5] proposed an adaptive method, which can be viewed as a more sophisticated non-linear predictor.

In [7], the authors presented a method for designing more complex non-linear perfect reconstruction filter banks. This paper demonstrates the potential of this method by presenting some example non-linear filters for still image compression.

## 2. THEORY

In this paper we will consider a poly-phase or sub-sampled domain implementation of the filters, as shown in figure 3. We will assume that the down-sampling and up-sampling process are critically decimated and loss-less (in the sense that without any quantization or filtering, the original signal is recovered).

It is convenient to write  $\mathbf{Y}$  as the vector of input signals  $y_0 \dots y_{n-1}$ .  $\mathbf{T}$  and  $\hat{\mathbf{Y}}$  can be defined in the same way, as  $t_0 \dots t_{n-1}$  and  $\hat{y}_0 \dots \hat{y}_{n-1}$  respectively.

The system can now be written as:

$$\mathbf{T} = \mathbf{A}(\mathbf{Y}) \quad \hat{\mathbf{Y}} = \mathbf{B}(\mathbf{T}) \quad (1)$$

With linear filters, these equations are reduced to matrix multiplications with the poly-phase filters  $\mathbf{A}(z)$  and  $\mathbf{B}(z)$ .

For perfect reconstruction  $\mathbf{B}(\cdot)$  must be the inverse,  $\mathbf{A}^{-1}(\cdot)$ , of  $\mathbf{A}(\cdot)$  such that:

$$\mathbf{Y} = \mathbf{A}^{-1}(\mathbf{A}(\mathbf{Y})) \quad (2)$$

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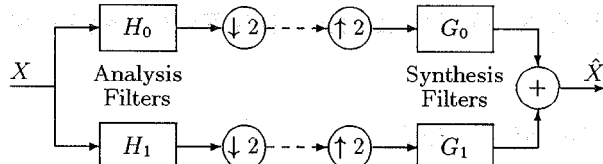


Figure 1: 2-band filter bank.

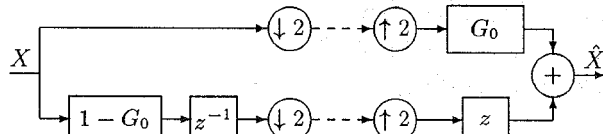


Figure 2: Half-band or interpolative system.

A block filter  $A(\cdot)$  which has a realizable inverse is termed as invertible. Thus the task of designing perfect reconstruction pairs is reduced to that of finding invertible block filters.

By considering the filter  $A(\cdot) = D(C(\cdot))$  we see that if  $C(\cdot)$  and  $D(\cdot)$  are invertible then  $A(\cdot)$  has an inverse  $A^{-1}(\cdot) = C^{-1}(D^{-1}(\cdot))$ , and thus  $C(\cdot)$  and  $D(\cdot)$  are a factorization of  $A(\cdot)$ .

Next we will introduce a relatively simple class of invertible block filter, which can be cascaded to form more complicated filters. Let  $\Phi_k(\mathbf{Y})$  be the set of signals  $y_n$  such that  $n \neq k$ . Then,  $\mathbf{T} = \mathbf{C}_k(\mathbf{Y})$  can be defined as:

$$\begin{aligned} t_n &= y_n & n \neq k \\ t_n &= y_n \oplus f_k(\Phi_k(\mathbf{Y})) & n = k \end{aligned} \quad (3)$$

with the inverse  $\mathbf{Y} = \mathbf{C}_k^{-1}(\mathbf{T})$  defined as:

$$\begin{aligned} y_n &= t_n & n \neq k \\ y_n &= t_n \ominus f_k(\Phi_k(\mathbf{T})) & n = k \end{aligned} \quad (4)$$

where the operators  $\oplus$  and  $\ominus$  are any invertible pair, such that  $(x \oplus y) \ominus y = x$ . There are many valid (linear and non-linear) operator pairs, which can be defined for various number types. The most useful example is, addition/subtraction defined over real, integer, or modulo- $m$  numbers. Other examples are, multiplication/division, exclusive-or, inclusive-and, etc.

The function  $f_k(\Phi_k(\mathbf{Y}))$  can be defined as any arbitrary linear or non-linear function, and can also include feedback to provide an IIR characteristic. Linear filters are achievable if the function  $f_k(\Phi_k(\mathbf{Y}))$  is linear, and the operator pair  $+, -$  is used, giving the poly-phase matrix  $\mathbf{C}_k(z)$  as the identity with added non-zero polynomials on the  $k$ th row.

The proposed structure is similar to the ladder structure of [8] with several significant extensions. For multi-band, non-linear systems the function  $f_k(\Phi_k(\mathbf{Y}))$  is more general than a linear sum of single argument terms

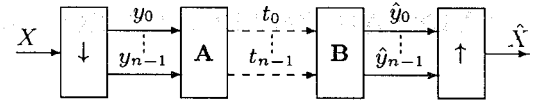


Figure 3:  $n$ -band filter bank in sub-sampled domain.

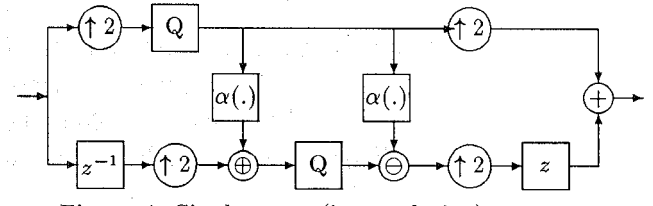


Figure 4: Single stage (interpolative) system.

$f_k(y_j)$ . Secondly, any invertible operator pair  $\oplus$  and  $\ominus$  may be used as opposed to just  $+$  and  $-$ . Thirdly, there is a wide flexibility in the choice of number system used, (eg. RGB triplets, and/or  $m$ -ary symbols).

### 3. EXAMPLE FOR STILL IMAGE COMPRESSION

The remainder of this paper presents an example non-linear filter bank for use within image coding.

Figure 4 shows a system with a single filtering stage,  $C_1(\cdot)$  (with  $f_1(\cdot)$  written as  $\alpha(\cdot)$ ). With the linear operators  $+$  and  $-$ , this structure corresponds to that of the half-band filters [5] or interpolative system [6]. The system is shown with the quantization of the upper sub-band placed prior to interpolation, which usually gives better results due to some compensation for quantization noise within the upper sub-band (see [5, 6]).

Figure 5 shows a two stage system  $C_0(C_1(\cdot))$ . The quantization of the lower sub-band could be placed before the filter  $\beta(\cdot)$ . It might be thought that this would allow some compensation for quantization noise. However, experiments have shown that such placement reduces the performance by limiting the functionality of the filter  $\beta(\cdot)$ .

A simple linear example of this system is, the odd length short kernel filters of [6], given by:  $H_0(z) = \frac{1}{8}(z^{-2} + 2z^{-1} + 6 + 2z + z^2)$  and  $H_1(z) = \frac{z^{-1}}{2}(-z^{-1} + 2 - z)$ . These are achieved with the filters,  $\alpha(z) = -\frac{1}{2}(1 + z^{-1})$  and  $\beta(z) = \frac{1}{4}(z + 1)$ . It is noted that this is an efficient implementation of these filters, since they can be realized with only 4 additions and 2 (power-of-two) multiplications, for each pair of samples.

Next, we will consider a quincunx sampled system (see figure 6). In this case the two sub-sampled signals are the two distinct co-sets, (shown as  $\bullet$  and  $\circ$ ). The  $z$  and  $z^{-1}$  (in figure 4) are replaced by the co-set vectors  $z_1 z_2$  and  $z_1^{-1} z_2^{-1}$ . We see that any sample  $q$  has four immediate neighbours from the complimentary sub-band  $(p_1, p_2, p_3, p_4)$ . A simple pair of linear

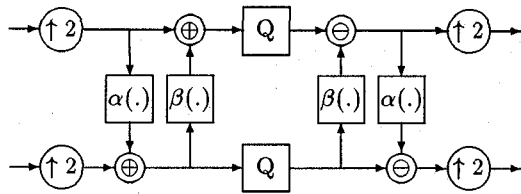


Figure 5: Two stage system.

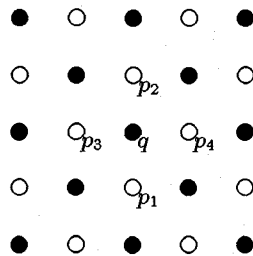


Figure 6: Quincunx sub-sampling lattice.

filters can be found with  $\alpha_1(\cdot) = -\text{mean}(p_1 \dots p_4)$  and  $\beta = \frac{1}{2}\text{mean}(p_1 \dots p_4)$ . The short impulse responses of the low-pass synthesis filter, implies low ringing artifacts, and makes the system well suited to image coding (see [2]). A simple and effective non-linear filter pair, can be achieved by replacing  $\alpha_1(\cdot)$  with  $\alpha_2(\cdot) = -\text{median}(p_1 \dots p_4)$ ; where the median of four points is defined to be the average of the middle two points after ordering.

#### 4. RESULTS

In order to demonstrate the effectiveness of these filters, we have simulated a quincunx systems for still image coding<sup>1</sup>, based on the single stage and two stage systems shown in figures 4 and 5; using the linear and non-linear filters  $\alpha_1$  and  $\alpha_2$  defined above.

Figure 7 shows the objective performance of each system. Table 1 gives results for coding at 0.3bpp. Figures 8 - 11 show the decoded images for both the single and two stage linear and non-linear systems, at 0.3bpp. More complicated linear filter banks [2] have also been tried. Although these can give slightly better objective results, they suffer more severely from ringing artifacts, which make them subjectively less acceptable.

These results show that the two stage non-linear filters give better peak signal to noise ratio (PSNR) than either the single stage linear or non-linear filters. Subjectively, the non-linear systems give much less visible quantization noise around sharp edges.

<sup>1</sup>Our experiments have been performed using the 256 x 256 'Lenna' image.

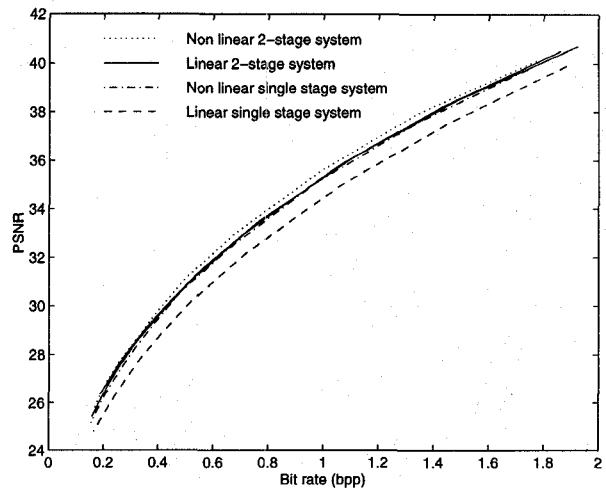


Figure 7: Objective performance comparison.

Filter bank	PSNR (dB)
Single stage linear	27.25
Two stage linear	28.31
Single stage non-linear	27.97
Two stage non-linear	28.33

Table 1: Results for 'Lenna' at 0.3 bpp.

#### 5. CONCLUSIONS

This paper has presented a new form of non-linear perfect reconstruction filter bank. The structures presented have a wide degree of flexibility allowing a wide choice of component functions, operator pairs and number systems to be used.

In order to demonstrate the potential of this approach, an example non-linear filter bank has been used for still image compression. The results show how the non-linear filters can provide both improved objective and subjective performance, compared to alternative linear and non-linear methods.

#### 6. REFERENCES

- [1] D. B. H. Tay and N. G. Kingsbury. Flexible design of multidimensional perfect reconstruction fir 2-band filters using transformations of variables. *IEEE Transactions on Image Processing*, 2:466-480, 1993.
- [2] O. Egger and W. Li. Subband coding of images using asymmetrical filter banks. *IEEE Transactions on Image Processing*, 4:478-485, 1995.
- [3] D. LeGall and A. Tabatabai. Sub-band coding of images using symetric short kernel filters and arithmetic coding techniques. In *International Con-*



Figure 8: Single stage linear system.



Figure 10: Two stage linear system.



Figure 9: Single stage non-linear system.



Figure 11: Two stage non-linear system.

*ference on Acoustic Speech and Signal Processing*, pages 761–764, 1988.

- [4] J. W. Woods. *Subband Image Coding*. Kluwer Academic Publishers, 1991.
- [5] O. Egger, W. Li, and M. Kunt. High compression image coding using an adaptive morphological subband decomposition. *IEEE proceedings*, 83:272–287, 1995.
- [6] P. Roos, M. A. Viergever, M. C. A. Vand Dijke, and J. H. Peters. Reversible intraframe compression of medical images. *IEEE Transactions on Medical Imaging*, 7:328–336, 1988.
- [7] D. W. Redmill and D. R. Bull. Non-linear perfect reconstruction critically decimated filter banks. *Electronics Letters*, 32:310–311, 1996.
- [8] F. A. M. L. Bruekers and A. W. M. van den Enden. New networks for perfect inversion and perfect reconstruction. *IEEE Journal on Selected Areas in Communications*, 10:130–137, 1993.